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**Transient Magnetohydrodynamic Liquid-Metal Flows in a Rectangular Channel with a Moving Conducting Wall**

by

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Bradford, Pennsylvania

and

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DTRC-88/017 Transient Magnetohydrodynamic Liquid-Metal Flows  
in a Rectangular Channel with a Moving Conducting Wall

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## NOMENCLATURE

$B_0$	Orientation of the dimensional externally applied, constant magnetic field
$C_1, C_2$	Fourier coefficients for transient fluid velocity in channel
$G$	Dimensionless pressure gradient in $x^*$ direction in rectangular channel $\frac{y_0^2}{\mu_f U_0} \frac{\partial P^*}{\partial x^*}$
$H$	Dimensionless induced magnetic field in $x^*$ direction in channel $H^*/U_0 \sqrt{\sigma \mu_f}$
$H^*$	Dimensional induced magnetic field in $x^*$ direction in rectangular channel
$h(y,z,t)$	Mathematical solution for the dimensionless induced magnetic field transient. The Fourier series solution is represented by

$$h(y,z,t) = \sum_{m=0,1,2,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} H_{mn}(t) \cos(m\pi y) \cos(n\pi z/(2k))$$

$$\text{where } H_{mn}(t) = \alpha C_1 e^{-P_1 t} + \beta C_2 e^{-P_2 t} \text{ and}$$

$$P_1 = [a(1 + Pm) + (1 - Pm)^2 a^2 - 4Pm(Mm\pi)^2]/(2Pm)$$

$$P_2 = [a + (Mm\pi)^2]/(P_1 Pm)$$

$H^{ss}(y,z)$	Mathematical solution for the dimensionless induced magnetic field for the steady state problem. The Fourier series is represented by
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$$H^{ss}(y,z) = \sum_{n=1,3,5,\dots}^{\infty} H_n(y) \cos(n\pi z/(2k))$$

$$\text{where } H_n(y) = [-g_1(e_1 + e_2)/d_1 + g_1(e_3 + e_4)/d_2]F$$

$$F = (4 \sin(n\pi/2))/((g_1 + g_2)n\pi)$$

$$g_1 = (M + \sqrt{M^2 + n^2 \pi^2 / k^2})/2$$

$$g_2 = (-M + \sqrt{M^2 + n^2 \pi^2 / k^2})/2$$

$k$	Dimensionless aspect ratio
-----	----------------------------

$M$	Hartmann number a dimensionless quantity $y_0 B_0 \sqrt{\sigma / \mu_f}$
-----	--

$P_m$  Magnetic Prandtl number a dimensionless quantity  $\sigma\mu_0\mu_f/\rho$

$u(y,z,t)$  Mathematical solution for the dimensional fluid velocity transient. The Fourier series is represented by

$$u(y,z,t) = \sum_{m=0,1,2..}^{\infty} \sum_{n=1,3..}^{\infty} U_{mn}(t) \sin(m\pi y) \cos(n\pi z/2k)$$

where  $U_{mn}(t) = C_1 e^{-P_1 t} + C_2 e^{-P_2 t}$  and

$$P_1 = [a(1 + P_m) + (1 - P_m)^2 a^2 - 4P_m(Mm\pi)^2 / (2P_m)]$$

$$P_2 = [a + (Mm\pi)^2] / (P_1 P_m)$$

$U^{ss}(y,z)$  Mathematical solution for the dimensionless fluid velocity for the steady state Rayleigh problem. The Fourier series solution is represented by

$$U^{ss}(y,z) = \sum_{n=1,3,5..}^{\infty} U_n(y) \cos \frac{n\pi z}{2k}$$

where  $U_n(y) = [g_2(e_1 - e_2)/d_1 + g_1(e_3 - e_4)/d_2]F$

$$F = (4 \sin(n\pi/2)) / ((g_1 + g_2)n\pi)$$

$$g_1 = (M + \sqrt{M^2 + n^2\pi^2/k^2})/2$$

$$g_2 = (-M + \sqrt{M^2 + n^2\pi^2/k^2})/2$$

$t^*$  dimensional time

$U^*$  dimensional fluid velocity in  $x^*$  direction

$U_0$  dimensional velocity of top perfectly conducting wall

$x^*, y^*, z^*$  dimensional cartesian coordinates

$x, y, z$  dimensionless cartesian coordinates  $y^*/y_0, z^*/y_0$

$y_0$  dimensional height of rectangular channel

$2z_0$  dimensional width of rectangular channel

$*$  all dimensional variables are denoted with a superscript asterisk which is contrary to previous work



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$\sigma$	Electrical conductivity
$\mu_f$	Viscosity
$\alpha C_1, \alpha C_2$	Fourier coefficients for transient induced magnetic field
$\mu_0$	Permeability of a vacuum

## ABSTRACT

*The magnetohydrodynamic equations for temporal transients have been formulated and solved for a liquid metal flowing in a rectangular channel. The rectangular channel has a perfectly conducting moving top wall and a perfectly conducting stationary bottom wall in the presence of an applied external magnetic field aligned perpendicular to the conducting walls. The side walls of the channel are stationary insulators. Calculations show that the temporal transients of the fluid velocity and induced magnetic intensity comprise two exponentially decaying parts. One fast transient is believed to be associated with the propagation of Alfvén waves and the other a slow transient being the result of viscous and electrical diffusion. Curves of the transients are presented at several stations in the channel.*

## ADMINISTRATIVE INFORMATION

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## INTRODUCTION

The use of liquid metals for current collectors in homopolar motors and generators has led to the design of machines of superior performance. The steady state power losses have been studied in a model of a current collector comprising a liquid-metal rectangular channel with an applied external magnetic field and boundary conditions containing a combination of moving and fixed insulating and perfectly conducting walls.<sup>1</sup> The top moving wall and stationary bottom wall are perfect conductors. The side walls are insulators. The applied external magnetic field is perpendicular to the conducting walls.

In some applications of homopolar generators it becomes necessary not only to start and stop the machines but also to operate them under oscillating conditions. This could be the case in an application where a homopolar generator behaves as an extremely high energy capacitor. Therefore, one is interested in examining energy losses caused by these non-steady state phenomena.

Studies of non-steady state Couette flow in an external magnetic field are seldom found in the literature. There have been some papers which augmented the classical Rayleigh problem<sup>2</sup> with an external magnetic field in an electrically conducting medium.<sup>3-8</sup> The presence of Alfvén waves and slowly propagating waves was observed in detailed studies of the Rayleigh problem. Theory predicted the possible formation of switch-on shocks and current sheets to be



present in the Alfvén wave front under certain conditions. Currently, it would appear that a complete theoretical analysis and explanation of the various phenomena comprising the Rayleigh problem is lacking. This is partly caused by a waning interest in the field of magnetohydrodynamics in the past decades and the complexity of the mathematical solutions in these magnetohydrodynamic problems.

Our objective is to present a transient two-dimensional solution and some numerical results for one particular geometry of finite extent, shown in Fig. 1. The solution enables the investigation of the influence of the sidewalls on the velocity and the induced magnetic field that affects the joulean and viscous dissipation. The geometry chosen roughly corresponds to an axisymmetric liquid metal current collector having channel dimensions that are small compared to the radius of curvature of the current collector. The moving and fixed conducting walls correspond to the rotor and the stator, respectively. Following earlier work,<sup>1</sup> the orientation of the applied magnetic field or the applied induction,  $B_0$  (see Fig. 1) is chosen to generate the maximum eddy current. In addition, current parallel to the applied magnetic field produces no electromagnetic body force and need not be considered. Analytical expressions are derived for the induced magnetic field and the fluid velocity in terms of their spatial and temporal variations. The parameters of variation are applied magnetic induction, fluid viscosity, density and electrical conductivity, and the channel aspect ratio. Numerical results showing the buildup of the velocity and induced magnetic field transients are presented for several cases.

### GENERAL TRANSIENT MAGNETOHYDRODYNAMIC EQUATIONS

Consider the case of laminar flow of an incompressible, uniformly conducting fluid of density  $\rho$ , viscosity  $\mu_f$ , and conductivity  $\sigma$  flowing in a rectangular duct of width  $2z_0$  and height of  $y_0$  with a uniform external magnetic induction,  $B_0$  applied in the  $y$  direction as shown in Fig. 1. The flow is driven by the sudden movement of the top conductor. This conductor, initially at rest, is assumed to attain a constant velocity at the onset of the transient calculation and maintains that velocity. It is assumed that the pressure gradient in the fluid is zero to simplify the solutions. Such would not be the case if there were any obstructions in the channel that limited the net flow of fluid. It is assumed that no secondary flow is generated during the transient, and there is no variation in the duct cross section or distortion of the applied induction in accordance with the assumption of a fluid velocity and an induced magnetic field intensity in the  $x$  direction only. The equation for the fluid velocity in the  $x$  direction is given by

$$\rho \frac{\partial U^*}{\partial t^*} = \mu_f \frac{\partial^2 U^*}{\partial y^{*2}} + \mu_f \frac{\partial^2 U^*}{\partial z^{*2}} + B_0 \frac{\partial H^*}{\partial y^*} - \frac{\partial P^*}{\partial x^*} \quad (1)$$

and the induced magnetic intensity in the  $x$  direction is given by the magnetic diffusion equation as

$$\sigma \mu_0 \frac{\partial H^*}{\partial t^*} = \frac{\partial^2 H^*}{\partial y^{*2}} + \frac{\partial^2 H^*}{\partial z^{*2}} + \sigma B_0 \frac{\partial U^*}{\partial y^*} \quad (2)$$

where all dimensional variables are denoted with a superscript asterisk, contrary to earlier work.<sup>1,8</sup> Equation (1) is the two-dimensional Navier-Stokes equation including an electromagnetic body force and Eq. (2) is the magnetic diffusion equation obtained by combining both of Maxwell's curl equations and the electrical constitutive equation for a moving conducting medium. All equations are in the SI or RMKSA system of units.

### RECTANGULAR CHANNEL WITH MOVING WALL

The problem solved in this paper considers a rectangular channel (see Fig. 1) filled with a liquid metal in a uniform magnetic field,  $B_0$ . The top wall is assumed to be perfectly conducting and is initially at rest. At time  $t^* = 0$  its velocity jumps in a unit step to  $U_0$ , where it remains constant. The bottom wall is stationary and is also an ideal conductor while the side walls parallel to the applied external magnetic induction field are stationary electrical insulators. To make the handling of the equations easier we define a number of convenient and physically meaningful dimensionless quantities. The dimensionless quantities are not denoted with asterisks as in previous work and are defined as

$$U = U^*/U_0, y = y^*/y_0, z = z^*/y_0, t = t^*/(y_0^2 \sigma / \mu_f),$$

$$H = H^*/U_0 \sqrt{\sigma \mu_f} \quad \text{and} \quad k = z_0/y_0, \quad \text{the Hartmann number}$$

$$M = y_0 B_0 \sqrt{\sigma / \mu_f}, \quad G = [y_0^2 / (\mu_f U_0)] \partial P^* / \partial x^*$$

the magnetic Prandtl number is given by  $Pm = \sigma \mu_0 \mu_f / \nu$ , where  $\mu_0$  is the permeability of a vacuum. Even though the Hartmann number was undefined in the magnetohydrodynamic Raleigh problem (because  $y_0$  was unbounded), it is the most important parameter in this problem. It is a measure of the ratio of ponderomotive force to the viscous force (in this interpretation it is assumed the ponderomotive force is of the order of  $U_0$ , certainly true for the electrical conductivities encountered in liquid metal current collectors). The magnetic Prandtl number is small in this problem but cannot be neglected entirely without causing great inaccuracy in the solutions. It can be discarded only in direct comparison with other larger quantities. The magnetic Prandtl number is a measure of the ratio of vorticity diffusion to magnetic diffusion. For liquid NaK the magnetic Prandtl number is of the order of about  $2 \times 10^{-6}$ . If liquid metal electrical superconductors were available, the magnetic Prandtl number would be very large and could cause interesting effects. To make the solutions much simpler we assume  $G = 0$ . Then Eqs. (1) and (2) become

$$\partial^2 U / \partial y^2 + \partial^2 U / \partial z^2 + M \partial H / \partial y = \partial U / \partial t \quad (3)$$

$$\partial^2 H / \partial y^2 + \partial^2 H / \partial z^2 + M \partial U / \partial y = Pm \partial H / \partial t \quad (4)$$

Here we note that the magnetic Prandtl number provides a coupling between the induced magnetic intensity and the fluid velocity that does not exist in the steady problem treated earlier.<sup>1</sup> Equations (3) and (4) must be solved simultaneously for  $U(y,z,t)$  and  $H(y,z,t)$  by any method which satisfies the

boundary and initial conditions while satisfying Eqs. (3) and (4). It is tempting to consider the use of the Laplace transform to eliminate time as a variable until the difficulty of the inversion of the transform is considered. Instead of using the Laplace transform method for solution of the partial differential equations, we have decided to seek a solution in terms of exponentially diminishing time functions. To do that we let

$$U(y,z,t) = U_{ss}(y,z) + u(y,z,t) \quad (5)$$

$$H(y,z,t) = H_{ss}(y,z) + h(y,z,t) \quad (6)$$

where  $U_{ss}$  and  $H_{ss}$  are the solutions to the steady flow problem and  $u(y,z,t)$  and  $h(y,z,t)$  are transients that eventually fade. When Eqs. (5) and (6) are substituted into Eqs. (3) and (4) there results for the transients  $u$  and  $h$

$$\partial^2 u / \partial y^2 + \partial^2 u / \partial z^2 + M \partial h / \partial y = \partial u / \partial t \quad (7)$$

$$\partial^2 h / \partial y^2 + \partial^2 h / \partial z^2 + M \partial u / \partial y = Pm \partial h / \partial t \quad (8)$$

#### BOUNDARY CONDITIONS

The no slip condition applies to the total solution for the velocity at all of the boundaries. In that case

$$U(y=1,z,t) = 1 \text{ implying that } u(y=1,z,t) = 0$$

$$U(y=0,z,t) = 0 \text{ implying that } u(y=0,z,t) = 0$$

$$U(y,z=\pm k,t) = 0 \text{ implying that } u(y,z=\pm k,t) = 0$$

Because there must be zero electric field in the rest frame of the perfect conductors for finite current flow, the normal derivative of the magnetic intensity is zero at the top and bottom perfect conductors. Currents generated in the top conductor and the fluid move in a loop (in the  $y$ - $z$  plane) bounded by the conductors and the side insulating walls. All the current generated by conductors moving in the applied external induction field returns primarily in the bottom perfect conductor. Thus by Ampere's law the induced magnetic intensity at the insulating walls is zero. In that case

$$\partial H / \partial y|_{y=1} = 0 \text{ implying that } \partial h / \partial y|_{y=1} = 0$$

$$\partial H / \partial y|_{y=0} = 0 \text{ implying that } \partial h / \partial y|_{y=0} = 0$$

for all values of  $z$  and  $t$ , and

$$H(y,z=\pm k,t) = 0 \text{ implying that } h(y,z=\pm k,t) = 0$$

These conditions hold only for the open circuit case with no net current between conductors.

Initially the total velocity and induced magnetic intensity are zero, which means the transient velocity and the transient induced magnetic intensity must be the negative of the steady solutions. In that case

$$u(y,z,t=0) = -U_{ss}(y,z) \text{ and}$$

$$h(y,z,t=0) = -H_{ss}(y,z)$$

#### MATHEMATICAL SOLUTION

To automatically satisfy all of the boundary conditions we choose solutions of the form

$$u(y,z,t) = \sum_{m=0,1,2..}^{\infty} \sum_{n=1,3..}^{\infty} U_{mn}(t) \sin(m\pi y) \cos(n\pi z/(2k))$$

$$h(y,z,t) = \sum_{m=0,1,2..}^{\infty} \sum_{n=1,3..}^{\infty} H_{mn}(t) \cos(m\pi y) \cos(n\pi z/(2k))$$

To satisfy the partial differential equations governing the solutions for  $u(y,z,t)$  and  $H(y,z,t)$ , the above assumed solutions are substituted into Eqs. (7) and (8). There results a pair of coupled time dependent differential equations in time given by

$$dU_{mn}/dt + \pi^2[m^2 + (n/(2k))^2]U_{mn} = -Mm\pi H_{mn} \quad (11)$$

$$PmdH_{mn}/dt + \pi^2[m^2 + (n/(2k))^2]H_{mn} = Mm\pi U_{mn} \quad (12)$$

Equations (11) and (12) are combined to yield a pair of second order time dependent differential equations for  $U_{mn}$  and  $H_{mn}$ . They are given by

$$Pmd^2/dt^2 + (1+pm)[(m\pi)^2 + (n\pi/(2k))^2]d/dt + (Mm\pi)^2 + [(m\pi)^2 + (n\pi/(2k))^2]^2 \begin{bmatrix} U_{mn} \\ H_{mn} \end{bmatrix} = 0 \quad (13)$$

We assume the solution to this equation is  $U_{mn}(t) = C e^{-Pt}$  whence  $P$  must be found from the roots to the equation

$$PmP^2 - (1+Pm)aP + (Mm\pi)^2 + a^2 = 0 \quad (14)$$

where  $a = \pi^2[m^2 + (n/(2k))^2]$ . Because the two solutions of Eq. (14) may differ greatly in magnitude we choose them to be of the form<sup>8</sup>

$$P_1 = [a(1 + Pm) + (1 - Pm)^2 a^2 - 4Pm(Mm\pi)^2]/(2Pm) \quad (15)$$

and

$$P_2 = [a + (Mm\pi)^2]/(P_1 Pm) \quad (16)$$

Thus the solutions to Eq. (13) are given in exponential form by

$$U_{mn}(t) = C_1 e^{-P_1 t} + C_2 e^{-P_2 t} \quad (17)$$

$$H_{mn}(t) = \alpha C_1 e^{-P_1 t} + \beta C_2 e^{-P_2 t} \quad (18)$$

where  $\alpha = (P_1 - a)/(Mm\pi)$  and  $\beta = (P_2 - a)/(Mm\pi)$ . Utilizing the aforementioned initial conditions yields

$$C_1 + C_2 = -U_{mn}^{ss} \quad (19)$$

$$\alpha C_1 + \beta C_2 = -H_{mn}^{ss} \quad (20)$$

having solutions of

$$C_1 = (U_{mn}^{ss} - \beta H_{mn}^{ss})/(\alpha - \beta) \quad (21)$$

$$C_2 = (-\alpha U_{mn}^{ss} + H_{mn}^{ss})/(\alpha - \beta) \quad (22)$$

where  $U_{mn}^{ss}$  and  $H_{mn}^{ss}$  are the Fourier coefficients of the steady solution<sup>1</sup> when it is expressed as a double Fourier series of the forms of Eqs. (9) and (10). To reduce the amount of tedious algebra resulting upon the invocation of the orthogonality of the sinusoidal functions we have tried to express the steady state results<sup>1</sup> as simply as possible. These solutions are given by

$$U^{ss}(y, z) = \sum_{n=1,3,5,\dots}^{\infty} U_n(y) \cos(n\pi z/(2k)) \quad (23)$$

and

$$H^{ss}(y, z) = \sum_{n=1,3,5,\dots}^{\infty} H_n(y) \cos(n\pi z/(2k)) \quad (24)$$

where

$$U_n(y) = [g_2(e_1 - e_2)/d_1 + g_1(e_3 - e_4)/d_2]F,$$

$$H_n(y) = [-g_1(e_1 + e_2)/d_1 + g_1(e_3 + e_4)/d_2]F,$$

$$F = (4\sin(n\pi/2))/((g_1 + g_2)n\pi),$$

$$g_1 = (M + \sqrt{M^2 + n^2\pi^2/k^2})/2$$

$$g_2 = (-M + \sqrt{M^2 + n^2\pi^2/k^2})/2$$

$$e_1 = e^{-g_1(1-y)}$$

$$e_2 = e^{-g_1(1+y)}$$

$$e_3 = e^{-g_2(1-y)}$$

$$e_4 = e^{-g_2(1+y)}$$

$$d_1 = (1 - e)^{-2g_1} \text{ and}$$

$$d_2 = (1 - e)^{-2g_2}.$$

The standard use of orthogonality yields expressions for  $U_{mn}^{ss}$  and  $H_{mn}^{ss}$  as given by:

$$U_{mn}^{ss} = -2 \int_0^1 U_n(y) \sin(m\pi y) dy \quad (25)$$

$$H_{mn}^{ss} = -2 \int_0^1 H_n(y) \cos(m\pi y) dy \quad (26)$$

The indicated integration yields

$$U_{mn}^{ss} = \frac{(2\cos(m\pi)m\pi(-g_1^2 + g_1g_2 - g_2^2 - m^2\pi^2))}{(g_1^2g_2^2 + g_1^2m^2\pi^2 + g_2^2m^2\pi^2 + m^4\pi^4)}$$

and

$$H_{mn}^{ss} = \frac{(2\cos(m\pi)g_1g_2(g_1 - g_2))}{(g_1^2g_2^2 + g_1^2m^2\pi^2 + g_2^2m^2\pi^2 + m^4\pi^4)}$$

The substitutions of the results of Eqs. (27) and (28) into Eqs. (21) and (22), and the quantities defined after Eq. (24) and considerable amounts of algebraic manipulation yields  $C_1$  and  $C_2$ , which are given by

$$\begin{aligned}
C_1 = & (16\cos(m\pi)\sin((n\pi)/2)mPm(a^2g_1^2Op - a^2g_1g_2Op + a^2g_2^2Op + a^2m^2Op\pi^2 \\
& - ag_1^2g_2MOp + ag_1^2Ra - 2ag_1^2 + ag_1g_2^2MOp - ag_1g_2Ra + 2ag_1g_2 \\
& + ag_2^2Ra - 2ag_2^2 + am^2\pi^2Ra - 2am^2\pi^2 - g_1^2g_2MRa - 2g_1^2M^2m^2\pi^2 \\
& + g_1g_2^2MRa + 2g_1g_2M^2m^2\pi^2 - 2g_2^2M^2m^2\pi^2 - 2M^2m^4\pi^4)/(n(a^2g_1^2g_2^2Op^2 \\
& + a^2g_1^2m^2Op^2\pi^2 + a^2g_2^2m^2Op^2\pi^2 + a^2m^4Op^2\pi^4 + 2ag_1^2g_2^2OpRa \\
& - 4ag_1^2g_2^2Pm + 2ag_1^2m^2Op\pi^2Ra - 4ag_1^2m^2\pi^2Pm + 2ag_2^2m^2Op\pi^2Ra \\
& - 4ag_2^2m^2\pi^2Pm + 2am^4Op\pi^4Ra - 4am^4\pi^4Pm - 4g_1^2g_2^2M^2m^2\pi^2Pm \\
& + g_1^2g_2^2Ra^2 - 4g_1^2M^2m^4\pi^4Pm + g_1^2m^2\pi^2Ra^2 - 4g_2^2M^2m^4\pi^4Pm \\
& + g_2^2m^2\pi^2Ra^2 - 4M^2m^6\pi^6Pm + m^4\pi^4Ra^2)) \quad (27)
\end{aligned}$$

and

$$\begin{aligned}
C_2 = & (8\cos(m\pi)\sin((n\pi)/2)m(a^2g_1^2Op^2 - 2a^2g_1^2OpPm - a^2g_1g_2Op^2 \\
& + 2ag_1g_2OpPm + a^2g_2^2Op^2 - 2a^2g_2^2OpPm + a^2m^2Op^2\pi^2 \\
& - 2a^2m^2Op\pi^2Pm + 2ag_1^2g_2MOpPm + 2ag_1^2OpRa - 2ag_1^2PmRa \\
& - 2ag_1g_2^2MOpPm - 2ag_1g_2OpRa + 2ag_1g_2PmRa + 2ag_2^2OpRa \\
& - 2ag_2^2PmRa + 2am^2Op\pi^2Ra - 2am^2\pi^2PmRa + 2g_1^2g_2MPmRa + g_1^2Ra^2 \\
& - 2g_1g_2^2MPmRa - g_1g_2Ra^2 + g_2^2Ra^2 + m^2\pi^2Ra^2)/(n(a^2g_1^2g_2^2Op^2 \\
& + a^2g_1^2m^2Op^2\pi^2 + a^2g_2^2m^2Op^2\pi^2 + a^2m^4Op^2\pi^4 + 2ag_1^2g_2^2OpRa \\
& - 4ag_1^2g_2^2Pm + 2ag_1^2m^2Op\pi^2Ra - 4ag_1^2m^2\pi^2Pm + 2ag_2^2m^2Op\pi^2Ra \\
& - 4ag_2^2m^2\pi^2Pm + 2am^4Op\pi^4Ra - 4am^4\pi^4Pm - 4g_1^2g_2^2M^2m^2\pi^2Pm \\
& + g_1^2g_2^2Ra^2 - 4g_1^2M^2m^4\pi^4Pm + g_1^2m^2\pi^2Ra^2 - 4g_2^2M^2m^4\pi^4Pm \\
& + g_2^2m^2\pi^2Ra^2 - 4M^2m^6\pi^6Pm + m^4\pi^4Ra^2)) \quad (28)
\end{aligned}$$

For documentation we give the expressions for  $\alpha C_1$  and  $\beta C_2$  which are used for the calculation of the induced magnetic intensity. These expressions are

$$\begin{aligned}
\alpha C_1 = & (8\cos(m\pi)\sin((n\pi)/2)(a^3g_1^2Op^2 - 2a^3g_1^2OpPm - a^3g_1g_2Op^2 \\
& + 2a^3g_1g_2OpPm + a^3g_2^2Op^2 - 2a^3g_2^2OpPm + a^3m^2Op^2\pi^2 \\
& - 2a^3m^2Op\pi^2Pm - a^2g_1^2g_2MOp^2 + 2a^2g_1^2g_2MOpPm + 2a^2g_1^2OpRa \\
& - 2a^2g_1^2Op - 2a^2g_1^2PmRa + 4a^2g_1^2Pm + a^2g_1g_2^2MOp^2 \\
& - 2a^2g_1g_2^2MOpPm - 2a^2g_1g_2OpRa + 2a^2g_1g_2Op + 2a^2g_1g_2PmRa \\
& - 4a^2g_1g_2Pm + 2a^2g_2^2OpRa - 2a^2g_2^2Op - 2a^2g_2^2PmRa + 4a^2g_2^2Pm \\
& + 2a^2m^2Op\pi^2Ra - 2a^2m^2Op\pi^2 - 2a^2m^2\pi^2PmRa + 4a^2m^2\pi^2Pm \\
& - 2ag_1^2g_2MOpRa + 2ag_1^2g_2MPmRa - 2ag_1^2M^2m^2Op\pi^2 \\
& + 4ag_1^2M^2m^2\pi^2Pm + ag_1^2Ra^2 - 2ag_1^2Ra + 2ag_1g_2^2MOpRa \\
& - 2ag_1g_2^2MPmRa + 2ag_1g_2M^2m^2Op\pi^2 - 4ag_1g_2M^2m^2\pi^2Pm - ag_1g_2Ra^2 \\
& + 2ag_1g_2Ra - 2ag_2^2M^2m^2Op\pi^2 + 4ag_2^2M^2m^2\pi^2Pm + ag_2^2Ra^2 - 2ag_2^2Ra \\
& - 2aM^2m^4Op\pi^4 + 4aM^2m^4\pi^4Pm + am^2\pi^2Ra^2 - 2am^2\pi^2Ra \\
& - g_1^2g_2MRa^2 - 2g_1^2M^2m^2\pi^2Ra + g_1g_2^2MRa^2 + 2g_1g_2M^2m^2\pi^2Ra \\
& - 2g_2^2M^2m^2\pi^2Ra - 2M^2m^4\pi^4Ra))/(\text{Mn}\pi(a^2g_1^2g_2Op^2 + a^2g_1^2m^2Op^2\pi^2 \\
& + a^2g_2^2m^2Op^2\pi^2 + a^2m^4Op^2\pi^4 + 2ag_1^2g_2OpRa - 4ag_1^2g_2Pm \\
& + 2ag_1^2m^2Op\pi^2Ra - 4ag_1^2m^2\pi^2Pm + 2ag_2^2m^2Op\pi^2Ra - 4ag_2^2m^2\pi^2Pm \\
& + 2am^4Op\pi^4Ra - 4am^4\pi^4Pm - 4g_1^2g_2^2M^2m^2\pi^2Pm + g_1^2g_2^2Ra^2 \\
& - 4g_1^2M^2m^4\pi^4Pm + g_1^2m^2\pi^2Ra^2 - 4g_2^2M^2m^4\pi^4Pm + g_2^2m^2\pi^2Ra^2 \\
& - 4M^2m^6\pi^6Pm + m^4\pi^4Ra^2))
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
\beta C_2 = & (8\cos(m\pi)\sin((n\pi)/2)(-a^3g_1^2Op^2 + 2a^3g_1^2OpPm + a^3g_1g_2Op^2 \\
& - 2a^3g_1g_2OpPm - a^3g_2^2Op^2 + 2a^3g_2^2OpPm - a^3m^2Op^2\pi^2 \\
& + 2a^3m^2Op\pi^2Pm - 2a^2g_1^2g_2MOpPm - 2a^2g_1^2OpRa + 2a^2g_1^2Op \\
& + 2a^2g_1^2PmRa - 4a^2g_1^2Pm + 2a^2g_1g_2^2MOpPm + 2a^2g_1g_2OpRa
\end{aligned}$$



$$\begin{aligned}
& - 2a^2g_1g_2Op - 2a^2g_1g_2PmRa + 4a^2g_1g_2Pm - 2a^2g_2^2OpRa + 2a^2g_2^2Op \\
& + 2a^2g_2^2PmRa - 4a^2g_2^2Pm - 2a^2m^2Op\pi^2Ra + 2a^2m^2Op\pi^2 \\
& + 2a^2m^2\pi^2PmRa - 4a^2m^2\pi^2Pm - 2ag_1^2g_2MPmRa + 4ag_1^2g_2MPm \\
& + 2ag_1^2M^2m^2Op\pi^2 - 4ag_1^2M^2m^2\pi^2Pm - ag_1^2Ra^2 + 2ag_1^2Ra \\
& + 2ag_1g_2^2MPmRa - 4ag_1g_2^2MPm - 2ag_1g_2M^2m^2Op\pi^2 \\
& + 4ag_1g_2M^2m^2\pi^2Pm + ag_1g_2Ra^2 - 2ag_1g_2Ra + 2ag_2^2M^2m^2Op\pi^2 \\
& - 4ag_2^2M^2m^2\pi^2Pm - ag_2^2Ra^2 + 2ag_2^2Ra + 2aM^2m^4Op\pi^4 \\
& - 4aM^2m^4\pi^4Pm - am^2\pi^2Ra^2 + 2am^2\pi^2Ra + 4g_1^2g_2M^3m^2\pi^2Pm \\
& + 2g_1^2M^2m^2\pi^2Ra - 4g_1g_2^2M^3m^2\pi^2Pm - 2g_1g_2M^2m^2\pi^2Ra + 2g_2^2M^2m^2\pi^2Ra \\
& + 2M^2m^4\pi^4Ra)/(Mn\pi(a^2g_1^2g_2^2Op^2 + a^2g_1^2m^2Op\pi^2 + a^2g_2^2m^2Op\pi^2 \\
& + a^2m^4Op\pi^4 + 2ag_1^2g_2^2OpRa - 4ag_1^2g_2^2Pm + 2ag_1^2m^2Op\pi^2Ra \\
& - 4ag_1^2m^2\pi^2Pm + 2ag_2^2m^2Op\pi^2Ra - 4ag_2^2m^2\pi^2Pm + 2am^4Op\pi^4Ra \\
& - 4am^4\pi^4Pm - 4g_1^2g_2^2M^2m^2\pi^2Pm + g_1^2g_2^2Ra^2 - 4g_1^2M^2m^4\pi^4Pm \\
& + g_1^2m^2\pi^2Ra^2 - 4g_2^2M^2m^4\pi^4Pm + g_2^2m^2\pi^2Ra^2 - 4M^2m^6\pi^6Pm \\
& + m^4\pi^4Ra^2)) \tag{30}
\end{aligned}$$

The coefficients of the time functions in the exponentials are given by

$$\begin{aligned}
P1 = & (\pi i \sqrt{64M^2k^4m^2Pm - 16k^4m^4\pi^2Pm^2 + 32k^4m^4\pi^2Pm - 16k^4m^4\pi^2} \\
& - 8k^2m^2n^2\pi^2Pm^2 + 16k^2m^2n^2\pi^2Pm - 8k^2m^2n^2\pi^2 - n^4\pi^2Pm^2} \\
& + 2n^4\pi^2Pm - n^4\pi^2) + 4k^2m^2\pi Pm + 4k^2m^2\pi + n^2\pi Pm \\
& + n^2\pi)/(8kPm) \tag{31}
\end{aligned}$$

and

$$P_2 = \frac{(2\pi(4M^2k^2m^2 + 4k^2m^2 + n^2))/(i\sqrt{(64M^2k^4m^2P_m - 16k^4m^4\pi^2P_m^2 + 32k^4m^4\pi^2P_m - 16k^4m\pi^2 - 8k^2m^2n^2\pi^2P_m^2 + 16k^2m^2n^2\pi^2P_m - 8k^2m^2n^2\pi^2 - n^4\pi^2P_m^2 + 2n^4\pi^2P_m - n^4\pi^2) + 4k^2m^2\pi P_m + n^2\pi P_m + n^2\pi)}}{(32)} \quad (32)$$

$$M = \text{Hartmann number, } Op = 1 + P_m, Om = 1 - P_m \text{ and } Ra = \frac{\sqrt{Oma^2 - 4P_m(M\pi m)^2}}$$

Thus the transient parts of the velocity and magnetic intensity mathematical expressions are determined by Eqs. (9) and (10) and the auxiliary Eqs. (17), (18), and (27) through (32). In addition the above given definitions of  $g_1$ ,  $g_2$ ,  $Op$ ,  $Om$ , and  $Ra$  must be used.

#### LIMITING CASE SOLUTIONS

When the Hartmann number approaches zero the solution given above approaches the nonmagnetohydrodynamic limit of zero induced magnetic intensity and a purely hydrodynamic transient fluid dynamic velocity distribution. The complete nonmagnetohydrodynamic solution (including both the steady and the transient parts) is given by

$$U(y,z,t) = 4 \sum_{n=1,3,\dots}^{\infty} \frac{[\sin(n\pi/2)\cos(n\pi z/(2k))/[\pi n(1 - e^{-n\pi/k})]]}{[e^{n\pi(y-1)/(2k)} - e^{-n\pi(y+1)/(2k)}]} + 2 \sum_{m=1,2,3,\dots}^{\infty} \frac{m \cos(m\pi)\sin(n/2)\cos(n\pi z/(2k))e^{-t\pi^2[m^2 + (n/(2k))^2]}}{\pi^2 n [m^2 + (n/(2k))^2]} \quad (33)$$

In most laboratory experiments  $P_m$  is of the order of  $2 \times 10^{-6}$  and the expressions for  $P_1$  and  $P_2$  simplify to

$$P_1 = \pi^2[m^2 + n^2/(4k^2)]^2/P_m \quad (34)$$

and

$$P_2 = 1 + M^2m^2/[m^2 + n^2/(4k^2)] \quad (35)$$

where it is clear that the  $P_2$  terms contained in the second time function of Eqs. (9) and (10) do not aid in the convergence of the series, which is made up of

double summations. On the other hand the  $P_1$  terms, in addition to being very large, increase in size with the square of the indices  $m$  and  $n$ . From the inspection of these variables it is expected that the first term of the exponential solutions not only converges faster with index but also vanishes faster than the second term.

Exponential diminishing time solutions were assumed for the temporal variations as a matter of convenience. However, inspection of the radical contained in the solution of  $P_1$  indicates the possibility of complex roots. It is clear that oscillatory solutions exist when

$$M > (1 - P_m)[m^2 + n^2/(4k^2)]/(2m\pi P_m) \quad (34)$$

requiring extremely large Hartmann numbers in the terrestrial laboratory, assuming high temperature liquid metal superconductors are not available. If the magnetic Prandtl number can be increased, then some of the early terms in the summations may be damped sinusoids.

## RESULTS AND DISCUSSION

We present a few calculations of velocity and induced magnetic intensity transients, roughly covering the area of the channel. The calculations are done in double precision and require the summation of 50,000 terms for each point in space<sup>9</sup> ( $m$  and  $n$  ranging up to 1000 and 50 respectively). By the use of minimum-maximum method<sup>10</sup> only 400 terms are necessary for engineering accuracy. The optimization of the method for double series summations might further reduce the needed number of terms. To focus our attention upon the temporal variation of the velocity and the induced magnetic intensity, both quantities are normalized by dividing them by their final or steady state value. The steady state values are the same as those presented in an earlier paper.<sup>1</sup> By this technique the transients at different points in the channels can be compared most easily.

### TRANSIENTS

In Figs. 2, 4, 6 and 8 are shown normalized dimensionless velocity transients for an aspect ratio of unity a Hartmann number of 5 and a magnetic Prandtl number of  $2.54 \times 10^{-6}$ . In each case the velocity was normalized with respect to its steady state value. In each figure there are curves of normalized dimensionless velocity as a function of dimensionless time at  $y = 0.25, 0.5$  and  $0.75$ . A different value of  $z$  is used in each of the figures. In the case where  $z = 0$  (see Fig. 2), the transients are almost identical at all values of  $y$  presented. Inspection of Figs. 2, 4, 6 and 8 indicates the differences as the transients increase slightly when  $z$  increases from the channel center to the insulating walls and the risetime of the transient also increases slightly. The risetime for the velocity in the cases presented here is roughly 0.1 units of dimensionless time. Also, for the velocity transients the exponential solution having the small time constant has small amplitude compared to the large time constant exponential solution. That is,  $C_1$  is less than  $C_2$ .

These curves are presented in Figs. 3, 5, 7 and 9 for the same parameters as used in the corresponding velocity figures. In each case the induced magnetic field was normalized with respect to its steady state value. In all cases the normalized dimensionless induced magnetic intensity rises rapidly from zero to an apparent initial value (see Fig. 3) and subsequently undergoes less rapid changes. The expansion of the dimensionless time scale shows that the normalized dimensionless induced magnetic intensity actually begins at a value of zero (as it should) and increases very rapidly at first. This is the small time constant exponential part of the solution and as it fades the long time constant exponential part of the solution takes over. The magnetic intensity initially builds up faster at stations nearer the moving plate. We feel that the fast part of these transients is due to the passing of Alfvén waves while the slow part is due to viscous and magnetic diffusion. The curves are similar for the different values of  $z$  and show a slight tendency to have a slower approach to the steady state as  $z$  increases.

### CONCLUSION

A formal solution to the transient magnetohydrodynamic Couette problem has been obtained and we have presented but few of the situations that could be examined. We have disclosed the presence of fast and slow transients and the possibility of oscillatory behavior in both the fluid velocity and the induced magnetic field. In future work we plan to investigate these phenomena and start-up losses.

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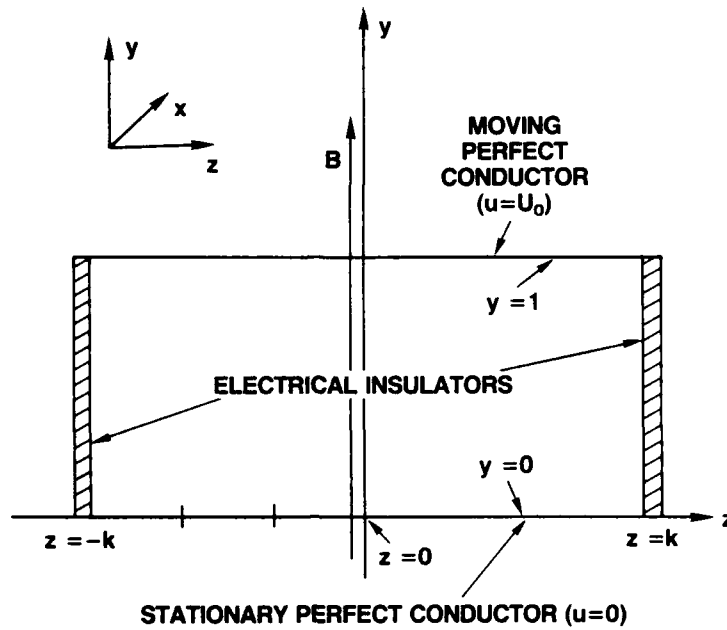


Fig. 1. Rectangular channel cross section.

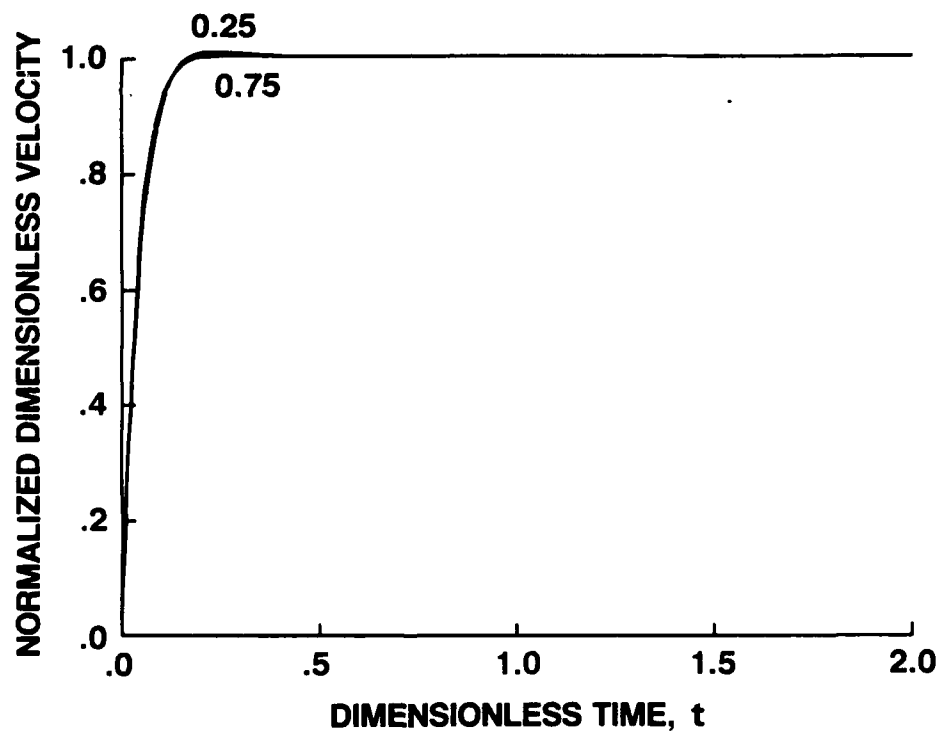


Fig. 2. Velocity transients at  $z = 0$  and  $y = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.

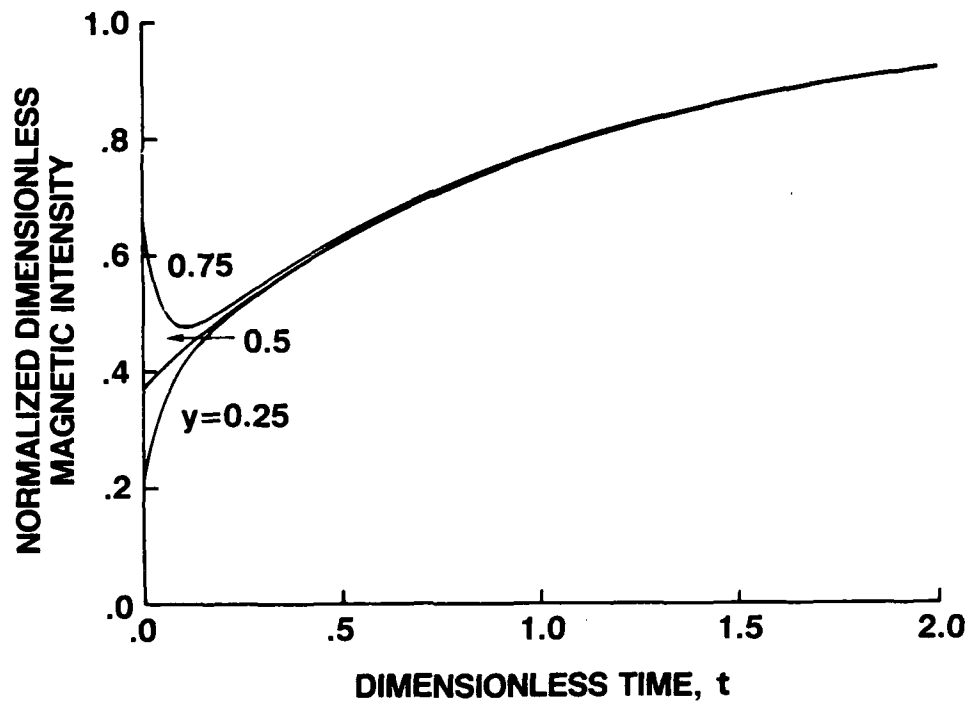


Fig. 3. Magnetic intensity transients at  $z = 0$ ,  $y = 0.25, 0.5$  and for  $k = 1$  and a Hartmann number of 5.

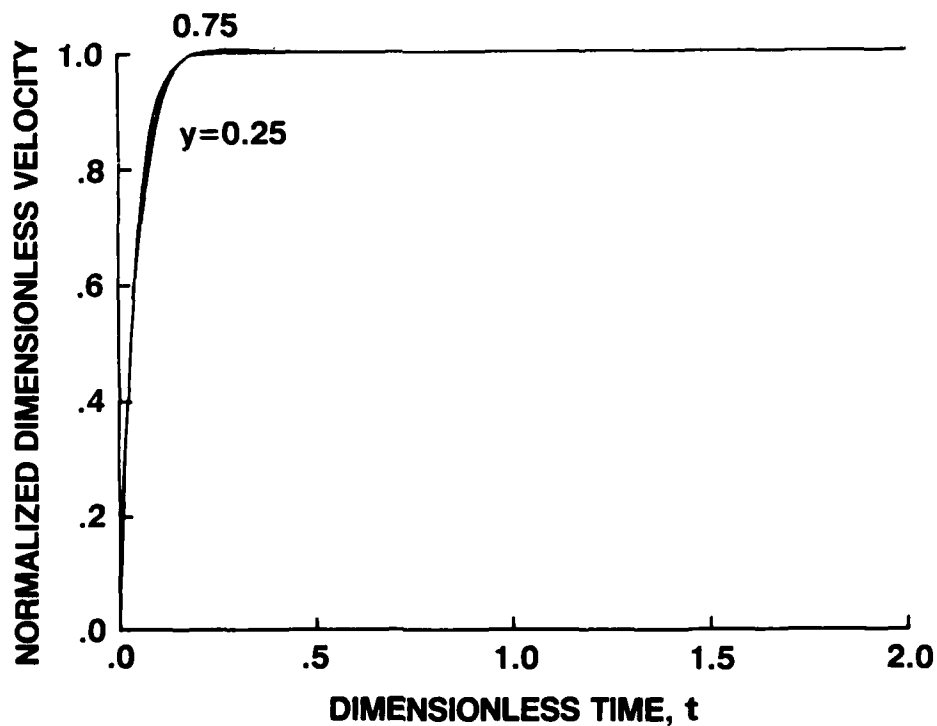


Fig. 4. Velocity transients at  $z = 0.25$ ,  $y = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.

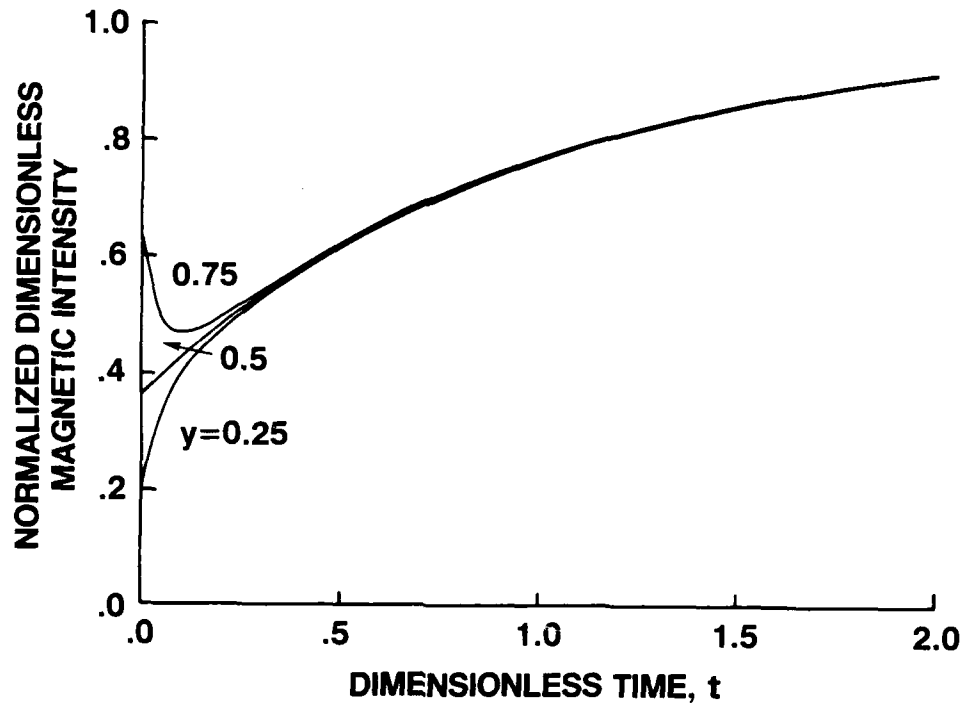


Fig. 5. Magnetic intensity transients at  $z = 0.25$ ,  $y = 0.25, 0.5$  and  $0.75$ , for  $k = 1$  and a Hartmann number of 5.

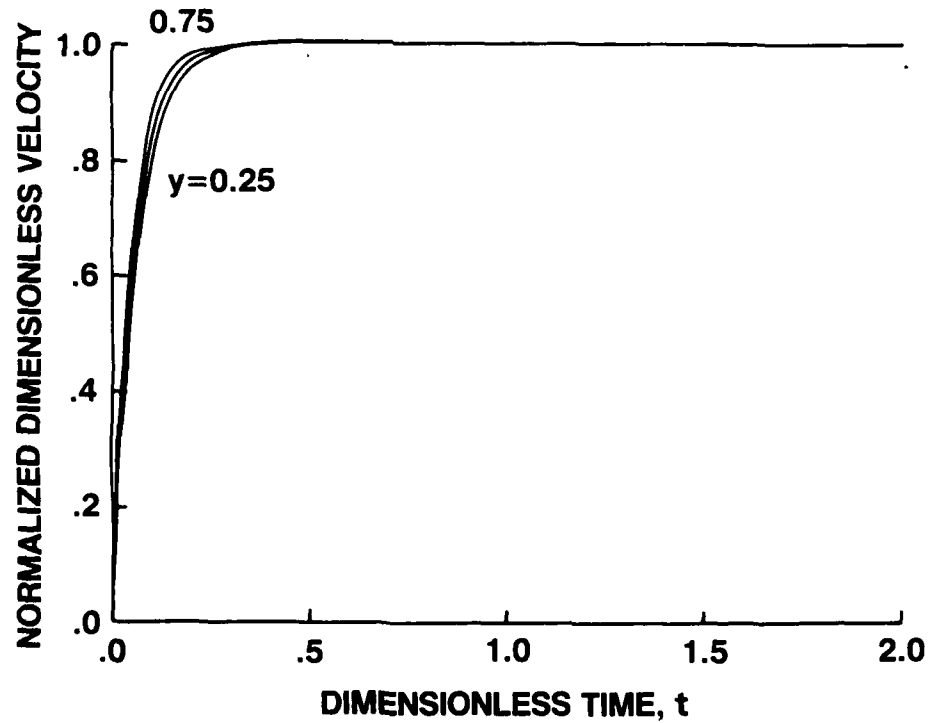


Fig. 6. Velocity transients at  $z = 0.5$ ,  $y = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.

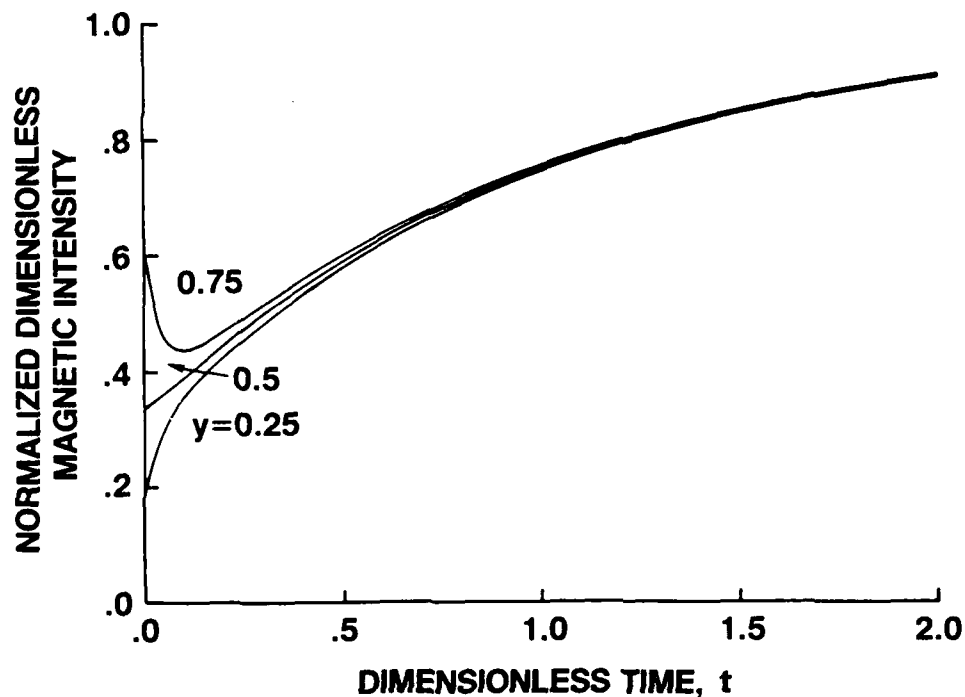


Fig. 7. Magnetic intensity transients at  $z = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.

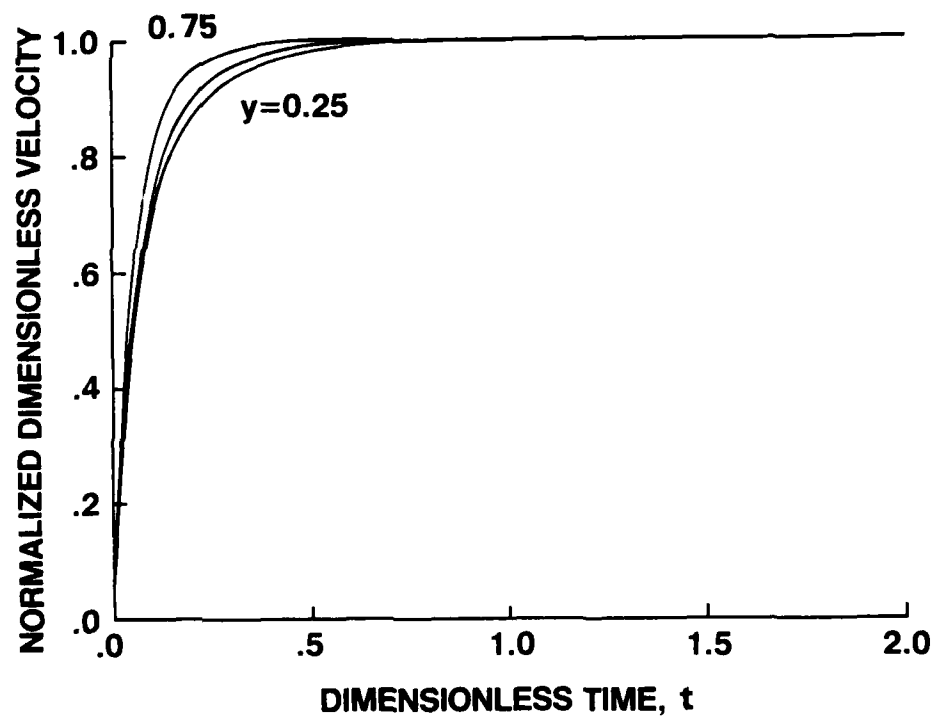


Fig. 8. Velocity transients at  $z = 0.75, y = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.



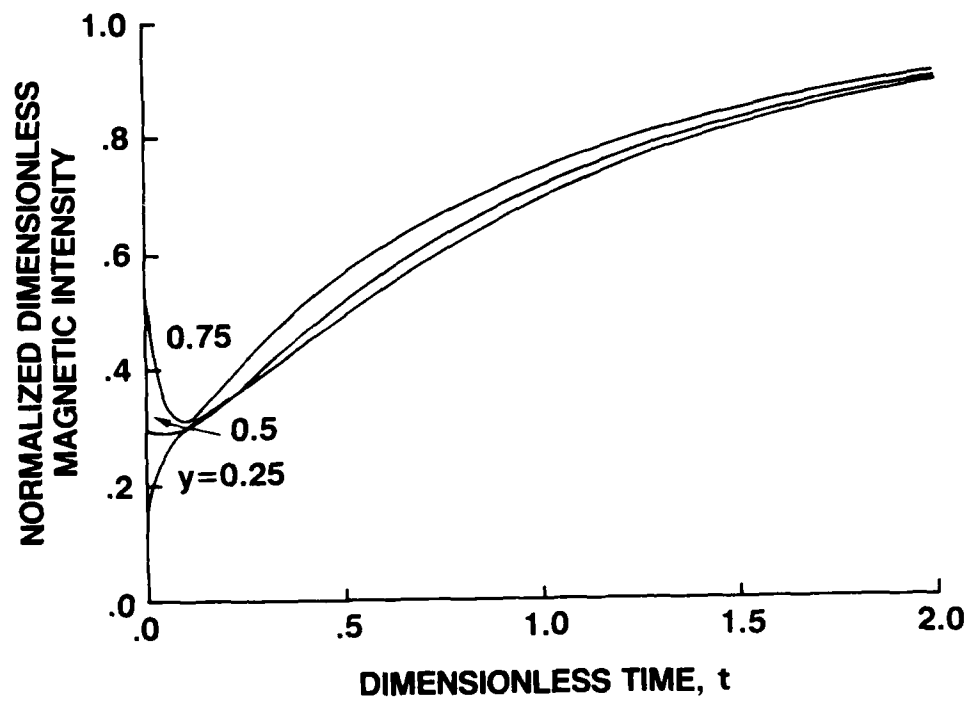


Fig. 9. Magnetic intensity transients at  $z = 0.75$ ,  $y = 0.25, 0.5$  and  $0.75$  for  $k = 1$  and a Hartmann number of 5.

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